Thema: Flächenmomente

Übung 8

Aufgabe 1

Ermitteln Sie für eine Hohlwelle mit einem Außendurchmesser von D = 50 mm und einem Innendurchmesser d von 40 mm die axialen und polaren Flächenmomente (I_x, I_y und I_p) sowie die entsprechenden Widerstandsmomente (W_x, W_y und W_p).

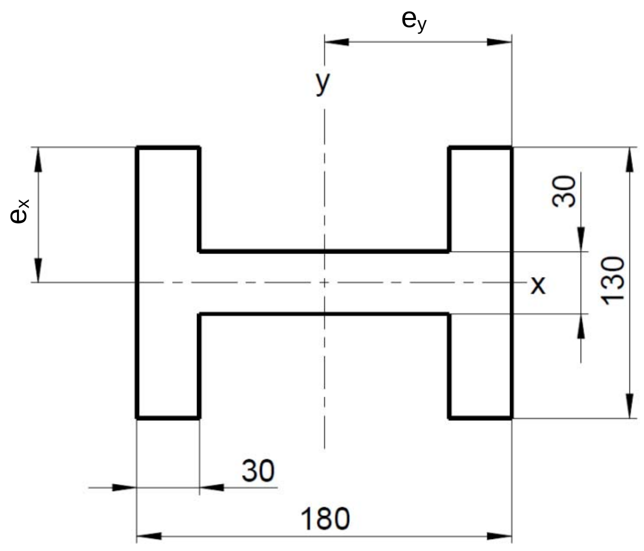
Aufgabe 2

Es ist der Querschnitt eines H-Profils dargestellt.

a) Ermitteln Sie die axialen Flächenmomente um die x-Achse und um die y-Achse.

b) Um welche Achse besitzt der Querschnitt die größere Biegesteifigkeit?

c) Um welche Achse würde der Querschnitt ausknicken?



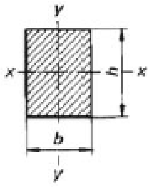
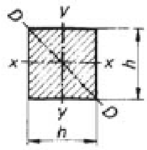
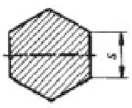
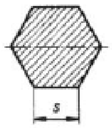
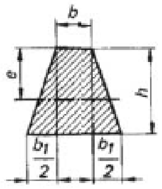
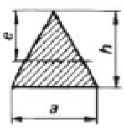
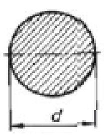
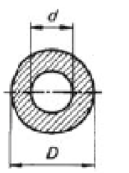
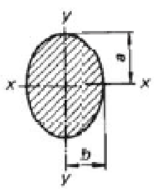
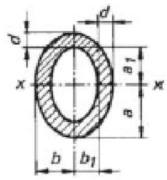
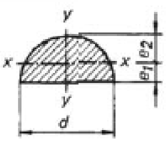
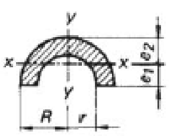
Aufgabe 3

Flächenmomente 2. Grades dürfen addiert und subtrahiert werden, wenn Teil und Gesamtschwerachse zusammenfallen. Widerstandsmomente dürfen auf keinen Fall addiert oder subtrahiert werden.

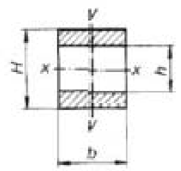
a) Das skizzierte H-Profil aus Aufgabe 2 soll entsprechend der Vorbemerkung zerlegt werden. Die Teilschwerachsen fallen mit der Gesamtschwerachse x-x und y-y zusammen.

b) Ermitteln Sie aus der Summe der Teilflächen die Flächenmomente I_x, I_y und die Widerstandsmomente W_x, W_y.

<u>Tabellen</u>: Axiale Flächenmomente I, Widerstandsmomente W sowie Trägheitsradius i für Biegung und Knickung

	$I_x = \frac{b h^3}{12}$ $I_y = \frac{h b^3}{12}$ $W_x = \frac{b h^2}{6}$ $W_y = \frac{h b^2}{6}$ $i_x = 0{,}289\,h$ $i_y = 0{,}289\,b$		$I_x = I_y = I_D = \frac{h^4}{12}$ $W_x = W_y = \frac{h^3}{6}$ $W_D = \sqrt{2}\,\frac{h^3}{12}$ $i = 0{,}289\,h$
	$I = \frac{5\sqrt{3}}{16} s^4 = 0{,}5413\,s^4$ $W = 0{,}5413\,s^3$ $i = 0{,}456\,s$		$I = \frac{5\sqrt{3}}{16} s^4 = 0{,}5413\,s^4$ $W = \frac{5}{8} s^3 = 0{,}625\,s^3$ $i = 0{,}456\,s$
	$I = \frac{6b^2 + 6bb_1 + b_1^2}{36\,(2b + b_1)} h^3$ $W = \frac{6b^2 + 6bb_1 + b_1^2}{12\,(3b + 2b_1)} h^2$ $e = \frac{1}{3}\,\frac{3b + 2b_1}{2b + b_1} h$		$I = \frac{a h^3}{36}$ $e = \frac{2}{3} h$ $W = \frac{a h^2}{24}$ $i = 0{,}236\,h$
	$I = \frac{\pi d^4}{64} \approx \frac{d^4}{20}$ $W = \frac{\pi d^3}{32} \approx \frac{d^3}{10}$ $i = \frac{d}{4}$		$I = \frac{\pi}{64}(D^4 - d^4)$ $W = \frac{\pi}{32}\,\frac{D^4 - d^4}{D}$ $i = 0{,}25\sqrt{D^2 + d^2}$
	$I_x = \frac{\pi a^3 b}{4}$ $I_y = \frac{\pi b^3 a}{4}$ $W_x = \frac{\pi a^2 b}{4}$ $W_y = \frac{\pi b^2 a}{4}$ $i_x = \frac{a}{2}$ $i_y = \frac{b}{2}$		$I_x = \frac{\pi}{4}(a^3 b - a_1^3 b_1)$ $I_x \approx \frac{\pi}{4} a^2 d\,(a + 3b)$ $W = \frac{I_x}{a} \approx \frac{\pi}{4} a d\,(a + 3b)$
	$I_x = 0{,}0068\,d^4$ $I_y = 0{,}0245\,d^4$ $W_{x1} = 0{,}0238\,d^3$ $W_{x2} = 0{,}0323\,d^3$ $W_y = 0{,}049\,d^3$ $i_x = 0{,}132\,d$ $e_1 = \frac{4r}{3\pi} = 0{,}4244\,r$		
	$I_x = 0{,}1098\,(R^4 - r^4) - 0{,}283\,R^2 r^2 \frac{R - r}{R + r}$ $W_{x1} = \frac{I_x}{e_1}$ $I_y = \pi \frac{R^4 - r^4}{8}$ $W_y = \pi \frac{(R^4 - r^4)}{8\,R}$ $W_{x2} = \frac{I_x}{e_2}$ $e_1 = \frac{2\,(D^3 - d^3)}{3\pi\,(D^2 - d^2)}$		

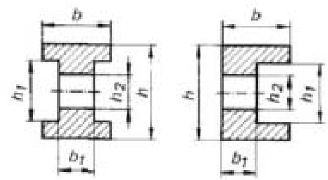
<u>Tabellen</u>: Axiale Flächenmomente I, Widerstandsmomente W sowie Trägheitsradius i für Biegung und Knickung



$$I_x = \frac{b}{12}(H^3 - h^3) \qquad I_y = \frac{b^3}{12}(H - h)$$

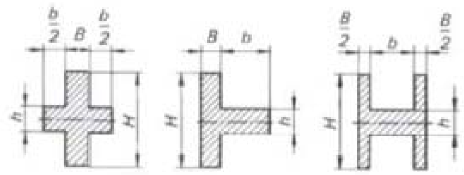
$$W_x = \frac{b}{6H}(H^3 - h^3) \qquad W_y = \frac{b^2}{6}(H - h)$$

$$i_x = \sqrt{\frac{H^3 - h^3}{12\,(H - h)}} \qquad i_y = 0{,}289\, b$$



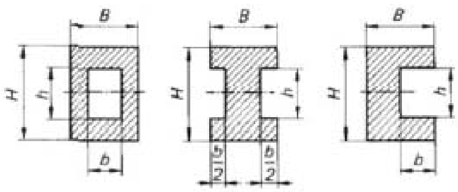
$$I = \frac{b\,(h^3 - h_1^3) + b_1\,(h_1^3 - h_2^3)}{12}$$

$$W = \frac{b\,(h^3 - h_1^3) + b_1\,(h_1^3 - h_2^3)}{6\,h}$$



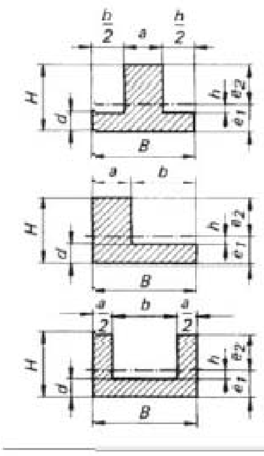
$$I = \frac{BH^3 + b\,h^3}{12}$$

$$W = \frac{BH^3 + b\,h^3}{6H}$$



$$I = \frac{BH^3 - b\,h^3}{12}$$

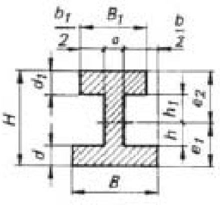
$$W = \frac{BH^3 - b\,h^3}{6H}$$



$$I = \frac{1}{3}\,(B e_1^3 - b\,h^3 + a\,e_2^3)$$

$$e_1 = \frac{1}{2} \cdot \frac{a\,H^2 + b\,d^2}{a\,H + b\,d}$$

$$e_2 = H - e_1$$

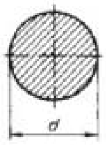
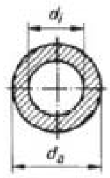
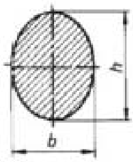
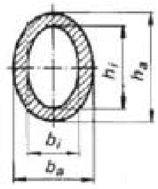
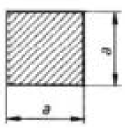
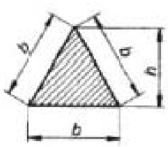


$$I = \frac{1}{3}\,(B e_1^3 - b h^3 + B_1 e_2^3 - b_1 h_1^3)$$

$$e_1 = \frac{1}{2} \cdot \frac{aH^2 + bd^2 + b_1 d_1\,(2\,H - d_1)}{a\,H + bd + b_1 d_1}$$

$$e_2 = H - e_1$$

<u>Tabellen</u>: Polare Flächenmomente I_p, Widerstandsmomente W_p für Torsion

	$W_p = \frac{\pi}{16} d^3 \approx \frac{d^3}{5}$	$I_p = \frac{\pi}{32} d^4 \approx \frac{d^4}{10}$	größte Spannung in allen Punkten des Umfangs
	$W_p = \frac{\pi}{16} \cdot \frac{d_a^4 - d_i^4}{d_a}$	$I_p = \frac{\pi}{32} (d_a^4 - d_i^4)$	größte Spannung in allen Punkten des Umfangs
	$W_p = \frac{\pi}{16} n b^3$ $\frac{h}{b} = n > 1$	$I_p \approx \frac{\pi}{16} \cdot \frac{n^3 b^4}{n^2 + 1}$	in den Endpunkten der kleinen Achse: $\tau_{t\,max} = \frac{M_T}{W_p}$ in den Endpunkten der großen Achse: $\tau_t = \frac{\tau_{t\,max}}{n}$
	$\frac{h_a}{b_a} = \frac{h_i}{b_i} = n > 1$ $\frac{h_i}{h_a} = \frac{b_i}{b_a} = \alpha < 1$ $W_p = \frac{\pi}{16} n b_a^3 (1 - \alpha^4)$	$I_p \approx \frac{\pi}{16} \cdot \frac{n^3}{n^2 + 1} \cdot$ $\cdot\, b_a^4 (1 - \alpha^4)$	in den Endpunkten der kleinen Achse: $\tau_{t\,max}$ in den Endpunkten der großen Achse: $\tau_t = \frac{\tau_{t\,max}}{n}$
	$W_p = 0{,}208\, a^3$	$I_p = 0{,}14\, a^4 = \frac{a^4}{7{,}1}$	in der Mitte der Seite: $\tau_{t\,max}$ in den Ecken: $\tau_t = 0$
	$W_p = 0{,}05\, b^3 = \frac{h^3}{7{,}5\sqrt{3}}$ $W_p = \frac{h^3}{13} = \frac{2 I_t}{h}$	$I_p = \frac{h^4}{15\sqrt{3}}$ $I_p = \frac{b^4}{46{,}2}$	in der Mitte der Seite: $\tau_{t\,max}$ in den Ecken: $\tau_t = 0$